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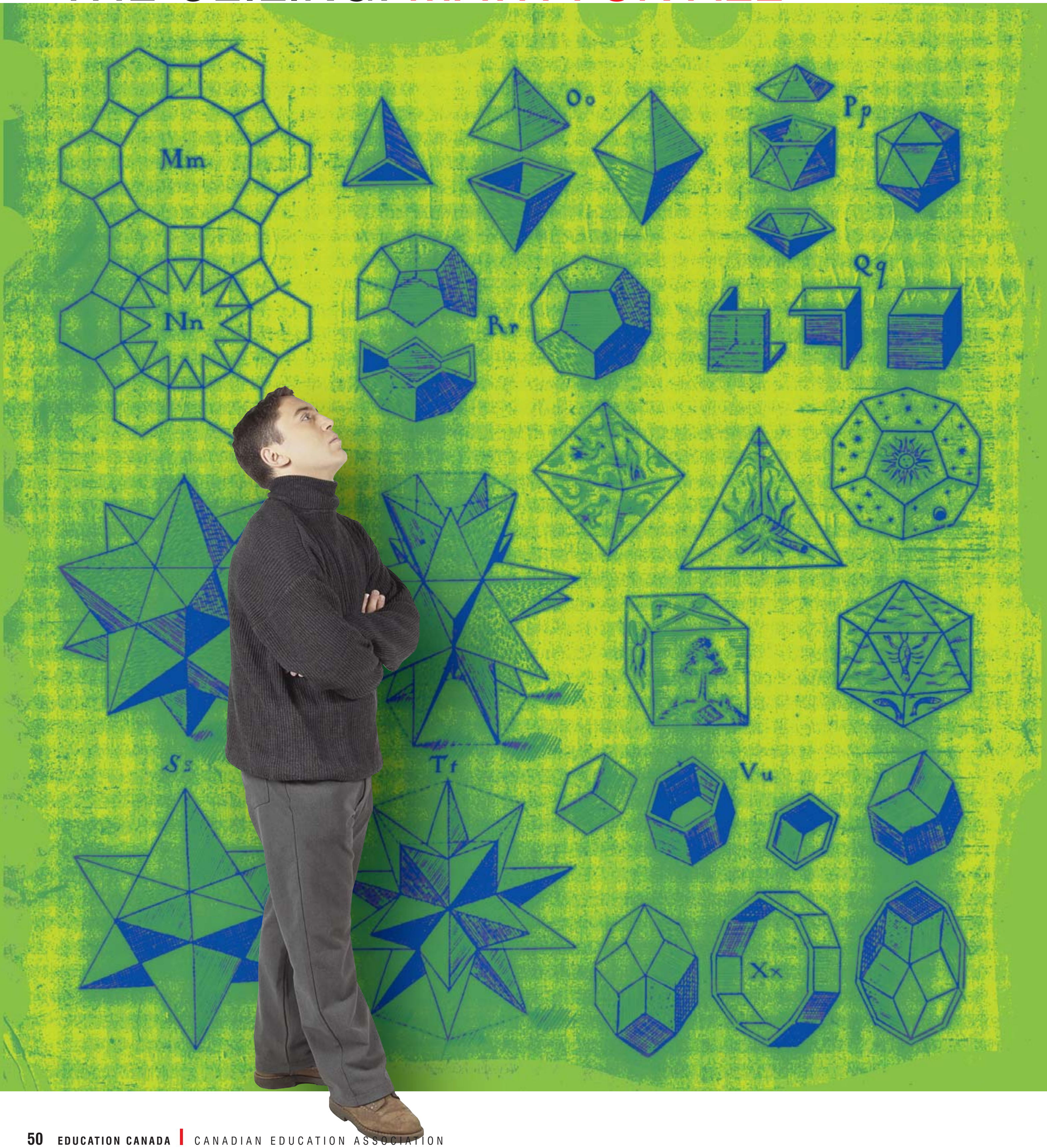
RAISING THE FLOOR AND LIFTING THE CEILING: MATH FOR ALL



EN BREF **Pour enseigner en vue de conférer la maîtrise des mathématiques, les enseignants doivent bâtir un environnement d'apprentissage procurant une solide base de connaissances détaillées. Les concepts de base qui sous-tendent les connaissances doivent être clairs. Voilà qui contraste vivement avec les pédagogies de type transmission et de type découverte. Les méthodes pédagogiques qui aident les élèves à acquérir la maîtrise des mathématiques combinent la formation en matière de concepts à la connaissance des méthodes. La conception universelle de l'apprentissage a été utilisée dans une classe comptant un nombre disproportionné d'élèves ayant des besoins particuliers. L'étude illustre que tous les élèves peuvent comprendre des concepts mathématiques difficiles lorsqu'on leur fournit des questions mathématiques valables, une évaluation formative dynamique, ainsi que des conversations d'approfondissement au cours desquelles les élèves doivent justifier leurs solutions et leur raisonnement.**

Math. The bane of my existence for as many years as I can count. I cannot relate it to my life or become interested in what I'm learning. I find it boring and cannot find any way to apply myself to it since I rarely understand it."[1]

PERHAPS MORE THAN any other discipline, the teaching of mathematics lends itself to procedural recipes where students memorize and duplicate procedures by rote: if it looks like this, do that to it. "If one believes that mathematics is mostly a set of procedures –rules and truths – and the goal is to help students become proficient executors of the procedures, then it is understandable that mathematics would be learned best by mastering the material incrementally, piece by piece."[2] Teaching practices that commonly flow from this procedural view are demonstration, repetition, and individual practice.

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In addition to being a misunderstanding of the discipline of mathematics itself, this belief also colours views about who can learn mathematics. Curricula and teaching practices are often based on what John Mighton calls a destructive ignorance "that leads us, even in this affluent age, to neglect the majority of children by educating them in schools in which only a small minority are expected to naturally love or excel at learning."[3] He insists that too many students lose faith in their own intelligence, and too much effort is directed at creating artificial differences between fast and slow, gifted and 'special', advanced and delayed.

Worse yet, procedural approaches to the teaching of mathematics that create problems of understanding and engagement are applied with even more vigour in remedial programs designed to help the very students for whom such practices did not work in the first place.

A growing number of researchers argue that we need new approaches to help students learn mathematics. "Today, mathematics education faces two major challenges: raising the floor by expanding achievement for all, and lifting the ceiling of achievement to better prepare future leaders in mathematics, as well as in science, engineering, and technology. At first glance, these appear to be mutually exclusive."[4] But are they? Is it possible to design learning that engages the vast majority of students in higher mathematics learning?

To answer these questions, I designed a research study to determine whether the principles of Universal Design for Learning (UDL) would result in increased student mathematical proficiency and achievement for all students in a typical Grade 7 classroom. A fairly recent educational innovation, UDL offers insights into proactively planning instruction that embraces the academic diversity characteristic of most ordinary classrooms. Would it be possible, in a regular classroom, to lift the ceiling and raise the floor?

The students in this study represented a fairly homogeneous population in terms of their demographic background, but the number of students with identified disabilities was 24.7 percent above the provincial average.

BUILDING MATH PROFICIENCY

Mathematical proficiency is developed through five interwoven and interdependent strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.[5] The Grade 7 students in this research study undertook a study of the concepts related to the geometry of particular shapes and solids by working through a number of tasks, each with a series of supporting subtasks designed to support the five strands. The idea that two- and three-dimensional objects, with or without curved surfaces, can be described, classified, and analyzed by their attributes was core to the central tasks. The following represents one of the tasks the students completed.

LINES, SHAPES, AND SPACES

Everywhere we see lines, shapes, and spaces. Working with a partner, you will need to:

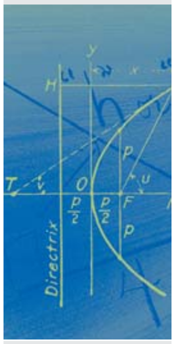
- Find lines, shapes, and spaces in the classroom, school or outside of the school.
- Take digital pictures of a variety of lines, shapes, and spaces you find.
- Download these pictures onto your laptop.
- Sort through your pictures and decide on at least two from each category, 1-D, 2-D, and 3-D, that you want to examine in great detail.
- Now, drag each of the pictures into a Word document. (Make sure you save each one with its own name and in a place you can both access.)
- Working with your partner, name, describe, analyze, and measure the various images you have collected.
- Select a way to present your work that best expresses how you understand the ideas in this task.

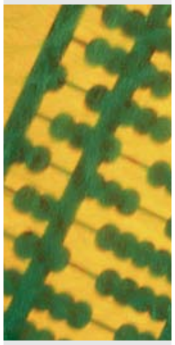
TEACHING FOR MATHEMATICAL PROFICIENCY REQUIRES TEACHERS TO DESIGN A LEARNING ENVIRONMENT THAT PROVIDES A SOLID FOUNDATION OF DETAILED KNOWLEDGE AND CLARITY ABOUT THE CORE CONCEPTS AROUND WHICH THAT KNOWLEDGE IS ORGANIZED.

I think it is important to note that students were not left alone to 'discover' the math for themselves. Rather, a series of lessons was designed to scaffold the student learning, ensuring that students uncovered and connected the underlying key concepts, worked through procedures related to measuring and calculating angles and arcs, length and perimeter, area and volume, congruence and similarity, and scale factors. They were asked to reason, to conjecture, and to justify conclusions.

As the teacher and I started to work with this particular task and the supporting subtasks, we quickly discovered that all the students needed additional guidance and instruction in how to work with the new demands placed on them. Their initial responses indicated that they were not accustomed to:

- working as a collaborative team
- listening to and building on each other's ideas
- challenging each other's thinking
- forwarding conjectures, and
- justifying their solutions





Teaching for mathematical proficiency requires teachers to design a learning environment that provides a solid foundation of detailed knowledge and clarity about the core concepts around which that knowledge is organized. The type of practice required to promote mathematical proficiency stands in sharp contrast to both transmission-type pedagogies and discovery-type pedagogies. Teaching practices that help students build mathematical proficiency combine concept formation with procedural fluency. In this way students gain a deep understanding of what they are doing and how to do it. It is also important for teachers to select or design robust, rich problems or tasks for students, problems that reach deep into fundamental mathematical ideas. In this way students experience why any of it matters.

DESIGNING MATH LEARNING FOR ALL

The research intervention designed for this study required us to establish a clear definition of mathematical proficiency, align that definition with findings from the learning sciences, identify key assumptions about the teaching of mathematics, specifically address students with identified learning needs, and evaluate the role of technology in the mathematics classroom.

UDL, which is grounded in emerging insights about brain development, learning, and digital media, was well suited to this task. Rose, Meyer, and Hitchcock have observed that the current disconnect between an increasingly diverse student population and our current 'one-size-fits-all' curriculum cannot produce the desired academic achievement gains expected in the 21st century.[6] They advance UDL as a means of focusing educational research, development, and practice on understanding diversity, technology, and learning. This new understanding calls for:

- *Multiple means of representation*, to give learners various ways of acquiring information and knowledge
- *Multiple means of expression*, to provide learners alternatives for demonstrating what they know
- *Multiple means of engagement*, to tap into learners' interests, offer appropriate challenges, and increase motivation

Conventionally, when students have difficulties learning mathematical concepts and skills outlined in the Programs of Study, modifications to instructional planning, strategies, pedagogy, resources, and support are provided. Generally, material thought to be too complex or difficult for them to master is broken down into smaller, fragmented pieces. Often these students are given work that appears easier or that requires increased repetitive practice and skill-building drill on these fragmented pieces, sometimes with the aid of classroom assistants and pullout programs. Practice and skill-building are, indeed, a necessary part of learning mathematics; but when students lose sight of how the individual fragments they are learning relate to the bigger math idea, then reasoning and understanding are forfeited.

The prevailing assumption is that mathematical difficulties lie in the inherent inability of the individual student to master the mandated outcomes in ways that are unproblematic for normal or regular students. That is, the problem lies in the student, not in the way the math idea is presented. Accommodations are provided to remediate these difficulties to the extent possible for each disabled learner – however disability is defined for that individual. In a nutshell, differences among learners are generally regarded as a problem to be remedied with modification to existing programs.

In this study, all students completed the same task; however the ways they went about it differed. Day-to-day instruction was personalized to meet the learning needs of each student. This meant that a math concept was represented in a variety of different ways until the students grasped the ideas related to the concept. In UDL, the term 'representation' refers to the ways teachers organize or formulate content for classroom instruction to present key ideas and concepts. For teachers, then, it is no longer enough to know mathematics for their own use; creating or finding appropriate representations requires them to know about the discipline in ways that make it accessible to students.

The following one-day lesson provides a glimpse into one of the several ways the teacher and I worked with representing the concept of a circle.

CIRCLES ARE MORE THAN ROUND THINGS

Initially the students thought I was just being difficult when I pushed them in the naming of circles. After all, it's round, so it must be a circle.

"Really? How can you be sure?" I asked. "What exactly is a circle?"

The teacher and I needed to design a variety of different subtasks through which the students would develop a more accurate definition of a circle. We insisted they do this, as *we had noticed that the majority of students were misunderstanding pi, diameter, and radius, which are all built upon a correct understanding of a circle. We asked some groups of students to work with lengths of string, others with compasses, still others with dynamic geometry software. They needed to compare these circles to the circles they had in the images they had already collected. Then they needed to formulate a conjecture about what was true about all circles.*

After approximately 40 minutes of small group and individual instruction the teacher and I brought the students together to create a more mathematical definition and description of a circle, which they were now able to do because the representations they worked with and the supporting conversations were personalized.

Mathematics defines circles in a precise way. The idea that a circle is a set of points in a plane that are all an equivalent distance from a given fixed point is not trivial. This definition gives rise to a number of important concepts. In mathematics, names are much more than labels to memorize.

BUILDING ASSESSMENT INTO THE LEARNING

Dynamic assessment was intentionally built into the instructional design of student learning from the beginning, providing learning scaffolds and feedback to the students. The following assessment rubric was provided to the students before they undertook the first task we called Lines, Shapes and Spaces.

Multiple means of expression and assessment enable teachers to assess what students currently know and to identify and plan the required next steps. Dynamic assessment aligns closely with teaching goals and methods. Assessment *for* learning is an integral part of the instructional design process. It was also a key aspect of the design of this research study. Constant assessment of students' developing proficiencies and their misconceptions guided the direction of the intervention on an on-going basis.

If assessment practices remain a one-size-fits-all method of sorting out ability hierarchies in the classroom rather than a guide to learning and instruction, they will give incomplete pictures of the multiple ways in which individuals develop their proficiencies. Well-designed, embedded, dynamic assessment practices have the potential to remove many of the current barriers to learning in the mathematics classroom.

Assessment for learning in this study involved:

- Building criteria for success with the students.
- Ongoing sustained dialogue with students. A great deal of effort went into examining the geometric mathematical territory so the teacher and I could engage students in dialogue around questions that were worthy mathematically and which could assist students in making connections, developing reasoning, and building mathematical proficiency.
- Clearly understood learning goals and criteria. An analytic trait rubric was designed, thoroughly discussed, and made available to the students before the study started. The students used it constantly throughout the study as a roadmap to success. Using both the criteria that we had collaboratively established and the rubric, the students always knew where they were, where they were

An Invitation: Lines, Shapes and Spaces

	Keep Working	Getting There	You've Got it	In the Flow
Image	Digital photo is blurred or not clear.	Digital photo is clear, however the image is vague.	Digital photo is clear with an identifiable image.	Digital photo is clear with a sharply focused image.
Naming the lines, shapes, and spaces	Uses student's own naming structure	Names lines, shapes, and spaces using a mix of standard mathematical conventions and improvised names	Accurately names lines, shapes, and spaces using standard mathematical conventions	Accurately names lines, shapes, and spaces and provides details about which 'family' the line, shape, or space is part of
Description of lines, shapes, and spaces	Provides a description using own words	Provides a description that uses a mix of mathematical terminology and own improvised description	Provides an accurate description using mathematical terminology	Provides a detailed description that helps to illuminate features of the line, shape, and space
Analysis on lines, shapes, and spaces	Unable to figure out the various lines, shapes and spaces that compose this figure	Discerns some of the properties that comprise this figure	Accurately discerns most of the properties that comprise this figure	Analysis of this figure helps to illuminate the properties, showing how these lines, shapes, and spaces relate to other figures
Measures lines, shapes, and spaces	Finds a way to measure some of the lines, shapes, and spaces	Finds different ways to measure the lines, shapes, and spaces	Accurately measures the figure in a variety of different ways	Accurately measures the figure using a variety of ways and measuring tools
Teamwork	Conflicts between team members that interfere with quality and production	Cooperative team work in which team members reinforce each others' learning	Effective team work in which team members build on and extend each others' ideas	Effective team work in which team members build on, extend, and provide feedback on each others' ideas

going, and what they needed to do to address the gap. From this rubric, students found proof of learning in their own work and set goals for next learning steps.

- Specific daily constructive feedback, both written and oral, from the teacher, from me, and from each other around identified criteria.
- Feedback from the dynamic geometry application. Students got immediate feedback from the computer program when a construction wasn't working as they had intended it to.

CONCLUSION

This study revealed that in a short time – four weeks – even in a classroom with a disproportionately large number of identified special needs students, it is possible to raise both ceiling and floor simultaneously. All students wrote a pre-test and post-test which consisted of four questions from the OECD's Programme for International Student Assessment (PISA) 2000, 2003 and 2006. All showed statistically significant improvement in achievement.

All students made gains in all five strands of mathematical proficiency. All students engaged with and understood difficult mathematical ideas when they were provided with worthy mathematical questions, dynamic formative assessment to guide their learning and the teacher's teaching, and deep, probing conversations that required students to justify their solutions and thinking.

Choosing to teach math in a way that ensures success for all becomes, then, a matter of policy. How badly do we want equality for all students, and are we prepared to weather the inevitable storms as we start to shake up the status quo from the highest levels right down to the playground?

SHARON FRIESEN is the President of The Galileo Educational Network and an Associate Professor in the Faculty of Education at the University of Calgary. She is the winner of the 2007 Education Prize from the Pacific Institute for the Mathematical Sciences for making a significant contribution to education in the mathematical sciences.

Notes

1 The Alberta Teachers Association (ATA). *Trying to Teach, Trying to Learn: Listening to Students* (Edmonton, AB: The Alberta Teachers Association, 2003), 28.

2 J. Stigler and J. Hiebert, *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom* (New York, NY: The Free Press.1999), 90.

3 John Mighton, *The End of Ignorance: Multiplying Our Human Potential* (Toronto, ON: Knopf Canada, 2007), 2.

4 "Do the Math: Cognitive Demand makes a Difference" (American Educational Research Association: *Research Points* 4, no. 2, 2006), 1.

5 J. Kilpatrick, J. Swafford and B. Findell, eds., *Adding It Up: Helping Children Learn Mathematics* (Washington, D.C.: National Academy Press, 2001).

6 D. Rose, A. Meyer and C. Hitchcock, eds., *The Universally Designed Classroom: Accessible Curriculum and Digital Technologies* (Cambridge, MA: Harvard University Press, 2005).