JOHN MIGHTON

USING MATH AS A SPRINGBOARD TO SUCCESS

Ten years ago, with the help of several friends, I started a tutoring club called JUMP (Junior Undiscovered Math Prodigy) Math in my apartment. The program was founded on a very lucky accident: I had asked the principal of a local school to send some students who were having trouble with math. She misunderstood and delivered the most challenged students in the school, including a number in special education who were performing far below grade level. Working with these children, and with the thousands of students I have taught since the JUMP Math program started, has been an inspiration. It has convinced me not only that all children can learn, but also that mathematics can be a powerful tool for social justice. The intellectual poverty that societies have imposed on the majority of children, out of ignorance of their real potential, is the deepest source of material poverty because it perpetuates social inequity.

I have come to believe that we need to re-examine many of our ideas about how children learn and about what they are capable of learning. I am convinced that, if we are ever to nurture the full potential of children, we must develop a model of education based on a deeper understanding of the brain.

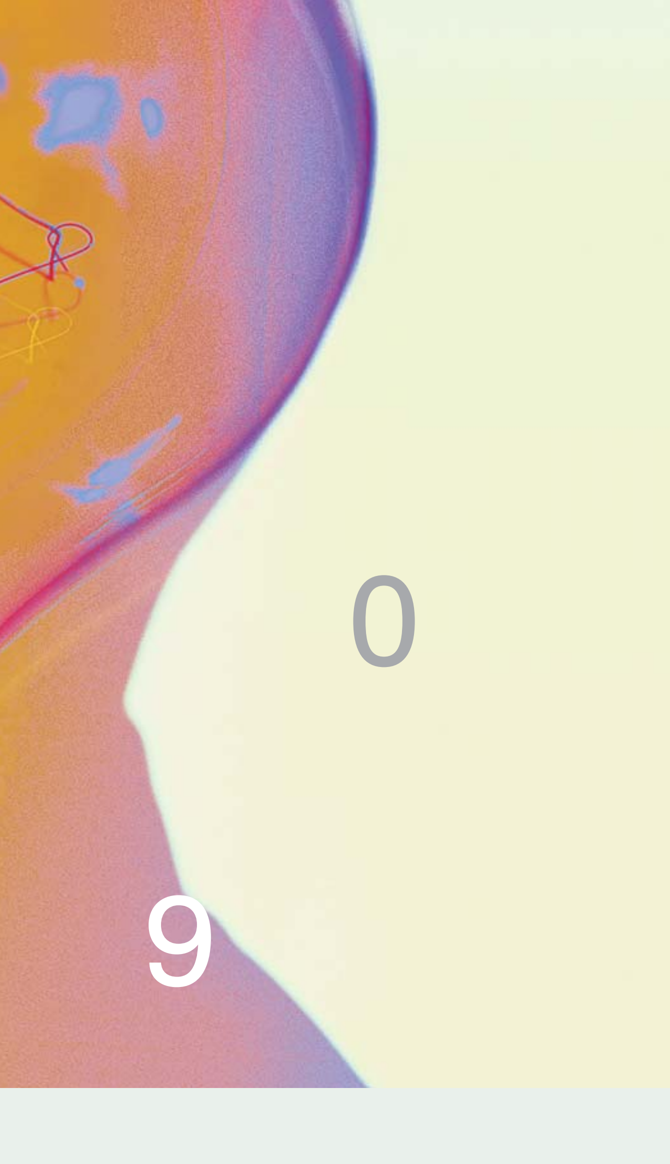
CURRENT RESEARCH

Our understanding of complex systems such as the brain has recently undergone a paradigm shift. Scientists now recognize that complex systems show emergent behaviour: new and unexpected properties of a system can emerge out of nowhere from a series of small changes. For example, if a chemist adds a reagent to a chemical solution one drop at a time, nothing may appear to be happening until – with the addition of just one more drop – the whole solution spontaneously changes colour.

In a series of groundbreaking experiments conducted over the last decade, scientists have shown that the human brain is much more 'plastic' and malleable than anyone had previously suspected, and that it can actually 'rewire' itself to repair damage or to develop new functions. Even older or impaired brains can develop new intellectual and creative abilities and can change their structure and their circuitry through rigorous cognitive training.

Consistent with these findings, the JUMP program has gathered a great deal of evidence over the past decade, through teacher testimonials and more rigorous large-scale pilots, that mathematical abilities can be nurtured in all students, including those who have learning disabilities or who have traditionally struggled at school. I am now convinced that new intellectual abilities can emerge in any student from a series of small advances, and that mathematics, rather than being the most difficult subject, is one in which a teacher can most easily add the drops of knowledge that can transform a student. And when students who traditionally struggle at school are allowed to succeed in math, they become far more engaged in learning in general. They also begin to develop the basic mental processing and logical skills, as well as the capacity to focus and stay on task, that they need to succeed at school.

Further, scientific evidence now suggests that children are born with roughly equal potential, and that what becomes of them is largely a matter of nurture, not nature. So how does so much potential ultimately disappear in so many children? Why do we observe such extreme differences in mathematical ability in students, even by the end of elementary school? "Instead of perpetually pondering the question, 'Why can't Johnny read?' perhaps educators should ask, 'Why should there be anything in the world



EN BREF Dans une série d'expériences inédites menées au cours de la dernière décennie, des scientifiques ont démontré que le cerveau humain est plus malléable qu'on le croyait et qu'il peut se « recâbler » pour réparer des dommages ou développer de nouvelles fonctions. En se fondant sur ces expériences, le programme JUMP a amassé une foule de renseignements selon lesquels les habiletés mathématiques peuvent être stimulées chez tous les élèves, y compris ceux qui ont traditionnellement du mal à réussir à l'école. Et si ces élèves se mettent à réussir en math, ils s'engagent beaucoup plus dans leur apprentissage en général. Ils commencent aussi à développer les habiletés élémentaires de traitement mental et de logique, ainsi que l'aptitude à se concentrer et à s'attarder sur une tâche, ce dont ils ont besoin pour réussir à l'école. En changeant leurs méthodes pédagogiques en fonction des nouvelles recherches sur la cognition, les enseignants peuvent donner de l'instruction rigoureuse permettant de produire des experts, tout en permettant à leurs élèves de faire des découvertes et d'apprendre en jouant.

that Johnny can't learn to do?'"[1]

I believe that the majority of people have trouble learning math because the theories of education on which our math programs are based do not take proper account of the conditions teachers face in real classrooms, nor do they take account of the new psychological models that explain how the brain works and how children learn. Fortunately there are signs that things are changing. A new American report, prepared by the President's Advisory Council, a body of influential educators and psychologists, advocates an increased focus on basic concepts and skills and a better balance between student-centered explorations (which are now the core of most math programs) and more rigorous teacher-directed instruction.

MATH SUCCESS FOR EVERYONE

A number of the recommendations of the President's Advisory Council coincide with the principles of the JUMP program, principles that I believe schools must take into account if they wish to teach math to all students.

Teachers must know their subject.

Many conscientious teachers readily admit that they are not entirely comfortable teaching math and that they struggled with the subject in school. In my workshops, I always ask teachers if they know why students are taught that, when multiplying a whole number by a fraction (say seven by two thirds), they should invert the fraction and multiply (multiply seven by three halves). Few have any idea why this works. It takes just a few minutes to explain an operation that has been a deep mystery to many of them.

A student who can only perform number operations with the help of a calculator will never be able to see patterns in numbers, or make predictions or estimates, or even know if the answers the calculator produces are correct. Similarly, teachers who attempt to teach mathematics without understanding the subject thoroughly will never be able to explain why various rules and operations work, or know how to help students who make mistakes or approach problems in original ways. And they will certainly never be able to inspire a love of math in their students. Fortunately, motivated teachers can quickly develop an expertise in math, as JUMP implementations have confirmed.

Students need periodic reviews and remediation.

At the beginning of every school year, teachers are faced with many students who have either forgotten, or didn't learn, much of the previous year's work. Most textbooks include only material that is mandated by the curriculum for a particular grade level, with a single review page at the beginning of a chapter, perhaps titled "Do You Remember?" The answer, for most students, is "No, I don't."

It's a mistake to assume that, just because children have managed to discover or understand mathematical operations or concepts, they will find it easy to recall those concepts and apply them to new situations months or years later. This assumption does not reflect my experience as a mathematician or as an educator. After months of practice, I discovered original (and rather elementary) algorithms in knot theory; but if you asked me how one of these algorithms works now, I would need several weeks of hard work to come up with the answer.

Research suggests that if teachers begin their year by providing whatever review and remediation is necessary, rather than sticking rigidly to the curriculum, they will ultimately cover far more curriculum by the end of the school year.

Students need rigorous guidance.

In "The Expert Mind," Philip Ross argues that logical and creative abilities, intuition, and expertise can be fostered in children through practice and rigorous instruction. But, as Ross points out, the research in cognition also shows that, to become an expert in a game like chess, for example, it is not enough to play without guidance or instruction. The kind of training in which chess experts engage – including playing small sets of moves over and over, memorizing

positions, and studying the techniques of the masters – appears to play a greater role in the development of ability than the actual playing out of a game. That is why, according to Ross, "enthusiasts can spend thousands of hours playing chess or golf or a musical instrument without ever advancing beyond the amateur level and why a properly trained student can overtake them in a relatively short time. It is interesting to note that the main training value of actual games is to point up weaknesses for future study."[2]

The idea that rigourous training produces experts more effectively than independent exploration runs counter to current educational practice. Over the past 20 years many educational theorists have claimed that if children are allowed to play with concrete materials – such as blocks and Cuisinaire rods – and to explore ideas with a little guidance from a teacher, they can turn themselves into experts in mathematics. According to this view, effective teachers can create conditions that will allow students, alone or in groups, to construct knowledge and make discoveries. As a 'guide on the side' rather than a 'sage on the stage', teachers shouldn't put too much emphasis on specific skills, nor should they fill up their students' heads with facts. The content is not as important as the way the student learns. This approach to teaching – usually known as discovery-based learning or inquiry-based learning – is now mandated, in one form or the other, in curricula across Canada and the United States.

The ideas behind discovery-based learning are not unreasonable in themselves. I believe very strongly, for instance, that teachers should allow students to discover things independently whenever they can. Even when I teach students in small steps, I always encourage them to take the steps themselves and to understand why they took them. This method of teaching, which I call guided discovery, is very different from rote learning. Even when students are capable of taking only the smallest steps, they are still actively engaged in discovering and constructing their knowledge of mathematics and are therefore led to understand at a deeper conceptual level. As students gain confidence through their successes and become more engaged in their work, I encourage them to work more independently and to attempt more open-ended problems.

As Paul Kirshner, John Sweller and Richard Clark argue:

> After half a century of advocacy associated with instruction using minimal guidance, it appears that there is no body of research supporting the technique. In so far as there is any evidence from controlled studies, it almost uniformly supports direct, strong instructional guidance rather than constructivist-based minimal guidance… Not only is unguided instruction normally less effective: there is also evidence that it may have negative results when students acquire misconceptions or incomplete or disorganized knowledge.[3]

The authors present several reasons why, based on the architecture of the brain, instruction with minimal guidance is not likely to be effective. They argue, for instance, that unguided instruction does not take account of the limitations of a student's working memory; the mind can retain only so much new information or so many component steps at one time.

Research in psychology and education confirms that younger students usually need a good deal of guidance and practice to learn mathematics, and they learn more easily when mathematical concepts and operations are broken down into the most basic elements of perception and understanding and introduced in a sequence of clear and logical steps. In the JUMP program, we introduce a variety of different ways of looking at or representing topics. For example, we present the concept of division through skip counting and repeated addition or subtraction, as the operation of forming a particular number of sets or a particular size of set, as the inverse of multiplication, through student investigations of various ideas (such as the meaning of the remainder), and through more formal presentation of standard algorithms.

Teachers should avoid overwhelming students with information.
Nobel prize winning cognitive scientist Herb Simon makes a case against a number of ideas that are currently popular in educational philosophy, including the idea that knowledge can always be communicated best in complex learning situations. He says:

> … a learner who is having difficulty with components can easily be overwhelmed by the processing demands of a complex task. Further, to the extent that many components are well mastered, the student wastes much less time repeating these mastered operations to get an opportunity to practice the few components that need additional effort.[4]

My experience confirms that students can easily be overwhelmed by too much information and that they learn best when working with only one or two concepts at a time, introduced in formats that are consistent and familiar.

Students need practice.
Adults think that repetition is tedious, so they seldom give children the practice they need to consolidate their understanding of skills and concepts. Anyone who has read a story to a child or watched a TV show like *Blues-Clues* (where the same episode is played five times a week) knows how much children love repetition. In *The First Idea*, a book on how intelligence evolved in humans, Stanley Greenspan and Stuart Shanker present evidence that infants develop cognitive abilities by learning to decipher subtle patterns in the voices, facial expressions and emotions of their caregivers. Even older children love to observe and create endless subtle variations on a pattern. According to Greenspan and Shanker, these activities help develop and consolidate neural pathways in the child's brain.[5]

Practice doesn't have to be painful for children, and repetition doesn't have to involve 'drill and kill'. If teachers are careful to introduce subtle variations into the work they assign, if they constantly raise the bar without raising it too far, and if they make learning into a game with different twists and turns, students will practice with real enjoyment.

THE BRAIN CAN'T REWIRE ITSELF OR REGISTER THE EFFECTS OF PRACTICE IF IT IS NOT ATTENTIVE. A CHILD'S BRAIN CAN ONLY BE ATTENTIVE IF THE CHILD IS CONFIDENT AND BELIEVES THAT THERE IS A POINT TO BEING ENGAGED.

Students need to master basic facts and operations.
It is a serious mistake to think that students who don't know number facts can get by in mathematics using a calculator or other aids. Students can certainly perform operations and produce numbers on a calculator, but if they don't have a sense of numbers, they will not be able to tell if their answers are correct, and they will not develop a talent for solving mathematical problems. To solve problems, students must be able to see patterns in numbers and make estimates and predictions about numbers. Trying to do mathematics without knowing basic number facts is like trying to play the piano without knowing where the notes are.

For all content areas, "practice allows students to achieve automaticity of basic skills – the fast, accurate and effortless processing of content information – which frees up working memory for more complex aspects of problems solving."[6]

Teachers must know how to assess what their students know.
The secret to bringing an entire class along at the same pace is to use 'continuous assessment'. When students are not able to keep up in a lesson, it is usually because they are lacking one or two basic skills or because they are being held back by a simple misconception. For instance, students who have trouble continuing or seeing patterns in number sequences are usually held back by a poor grasp of subtraction facts: they cannot determine quickly or accurately how much greater one term in a sequence is than the preceding term. But I have found that these students are able to take part in lessons on sequences when I have taken a few minutes before the lesson to teach them how to find the gap between numbers by counting up on their fingers.

The key to keeping track of what students know is to present ideas in steps, to assign small sets of questions or problems ('mini-quizzes' or tasks) that test their understanding of each step before the next one is introduced, and to insist that they present their answers in a way that allows teachers to spot mistakes easily. It can take a matter of seconds to check if a weaker student needs extra help and a matter of minutes to give students more practice with the one concept or skill that they need to master to do well on a quiz. Often I will only mark the answers of students who I think may be struggling, and then either move on or take up the quiz with the entire class.

The point of quizzes is not to rank students; they are an opportunity for student to show off what they know, to become more engaged in their work by meeting incremental challenges, and to experience the collective excitement that can sweep through a class when students experience success together.

The research that shows the brain is plastic also shows that it can't rewire itself or register the effects of practice if it is not attentive. A child's brain can only be attentive if the child is confident and believes that there is a point to being engaged. When students become convinced that they cannot keep up with the rest of the class, they are not attentive enough to consolidate new skills or develop new neural pathways. That is why it is so important to assess what students know, to give them the skills they need to take part in lessons, and to give them opportunities to show off by correctly answering questions and solving problems in front of the class.

CONCLUSION

These principles work. Hundreds of teachers have been able to significantly close the gap between their weaker and stronger students in math in the course of a year, showing that we have the knowledge we need to allow all children to reach their potential – and not just in math.

By changing their style of teaching to take account of new research in cognition, teachers could provide rigorous instruction of the sort that produces experts, while still allowing their students to make discoveries and play at learning. By changing the way they think about and assess ability, teachers could abolish artificial hierarchies from their classrooms and allow children to experience the joy of making discoveries and solving problems, both individually and collectively. As a society, we could simply refuse to settle for programs that do not nurture the potential of all children.

JOHN MIGHTON is a mathematician and playwright and is the Founding Director of JUMP Math, a registered charity that provides professional development for teachers as well as workbooks and Teacher's Guides for the full math curriculum from Grades 1 to 8. John works as a volunteer for JUMP and has donated all proceeds from the JUMP materials to the program. His recent book, *The End of Ignorance*, is published by Alfred A. Knopf, Canada.

Reports from several boards on JUMP implementation, and more information on the program, are available at www.jumpmath.org

Notes

1 Philip Ross, "The Expert Mind," *Scientific American*, July 2006, 44.
2 Ibid, 45.
3 Paul Kirshner, John Sweller and Richard Clark , "Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-Based, Experiential and Inquiry Based Teaching," *Educational Psychologist* 41 (2006): 75-86.
4 John R. Anderson, Lynne M. Reder and Herbert A. Simon, "Applications and Misapplications of Cognitive Psychology to Mathematics Education," *Texas Educational Review* (Summer 2000): 208.
5 Stanley Greenspan and Stuart Shanker, *The First Idea: How Symbols, Language and Intelligence Evolved from our Primate Ancestors to Modern Humans* (New York: Da Capo Press, 2006), 182-3.
6 President's Advisory Panel.