



# PROVE IT! Importance of Proof Thinking in Math Education

This story begins in an Ontario senior level math classroom where I was a student teacher. The desired outcomes for the calculus lesson in question were first coming to know and then learning to use the 'quotient rule' in computing the derivative, or rate of change, of a rational function, or relationship. It was in the process of coming to know this 'rule' that I began to formulate my own thoughts about the importance of proof thinking in mathematics.

We began with an investigation into the life of the man who brought us the trusted 'quotient rule', Sir Isaac Newton. Then, I gave students a handout purporting to be Sir Isaac's uncompleted proof to the quotient rule and challenged them to fill in blanks or, in some cases, entire lines of derivation. After the expected period of anguish, most relented, expressing something other than intellectual frustration. Students revealed an unwillingness to tackle the concept of proof thinking. My supervisor sympathized, applauded my efforts, and then informed me that the students were simply not used to such exercises. Furthermore, since the other two calculus teachers weren't going over proofs, such proof-based questioning couldn't appear on the standard test used across calculus sections. At that point, my thoughts about the importance of proof thinking began, and I seriously questioned why proof thinking isn't more central to math education.

In fact, the reasons for the inclusion – and in some cases, the centrality – of proof thinking in math education are manifold. While it is possible to speculate about political, institutional, and pedagogical factors leading to an educational environment that chooses not to embrace proof thinking – throughout the curriculum generally and in math particularly – I will concentrate on arguments for its unconditional integration. The role of proofs is to develop a capacity to think critically, both inside and – more importantly – outside the math classroom.

First, let it be clear that proof thinking in math education is more than simply working through arduous lines of centuries-old derivations to an end that asks students to recall those same lines during an assessment. Proof thinking harbours and begs for the sharpening of critical thinking. It does so in its refusal to apply anything without first understanding where it came from, no matter how painful the unearthing may be. Rather than beginning at the point of applying a theorem or rule to a problem, where it may

offer remedy, proof thinking starts at what lies behind the one line of ink that narrates such a formulaic tale. It starts there and refuses to stop moving backwards until it has completely and holistically quantified and qualified, scientifically and artistically, any piece of apparently valuable information.

Surely, such an ability to question material and to understand and prove its inner worth builds in students a sensibility that reaches far beyond mathematics – an acute responsiveness to the tendency to judge first and seek to understand next – or never – through acts of prejudice, hate, racism, oppression, injustice and other dehumanizing doings. If pupils experience, through proof thinking, the merit and correctness in first understanding where propositions (or formulae) originate, they can experience the same value in seeking to understand where propositions issued from those in power, or with hate-filled agendas, originate, and in doing so, will learn to reject any hints of sophisticated oppression or marginalization. Of course, the extension of those process-based abilities developed in the classroom to the realities of the playground and the marketplace ought to be highlighted by the conscientious educator. However, if proof thinking is encouraged seriously, holistically, often and in depth, it will automatically extend beyond the math classroom

And so, proof thinking is far more than memorizing lines; it inspires learners to seek authentic understandings which can then be used to solve problems. It is even possible to apply proof thinking as an ethical tool for correcting math's history. For example, a holistic approach to exploring proofs requires the teacher and student to understand the socio-cultural, political, and historical climate surrounding a given class of mathematicians. This holistic approach may reveal why some mathematicians were either encouraged or discouraged to construct new math

**AN ABILITY TO QUESTION MATERIAL AND TO UNDERSTAND AND PROVE ITS INNER WORTH BUILDS IN STUDENTS A SENSIBILITY THAT REACHES FAR BEYOND MATHEMATICS – AN ACUTE RESPONSIVENESS TO THE TENDENCY TO JUDGE FIRST AND SEEK TO UNDERSTAND NEXT.**



thought – and therefore either developed, or did not develop, proofs in support of that thought. Placing this anthropological lens on existing math thought in general, and proofs in particular, rightfully stretches the canvas upon which students come to know what math is. That is a daring but correct place to stand: to actually question whether our understanding of math constitutes the entire picture.

At this juncture, math is no longer just a diverse set of useful theorems and formulae, objective or otherwise, of which we may know only a proportion or a particular version. Math becomes highly problematized and exposed – as should all academic spheres – for the worldly baggage it carries, because no math was created in a cultural or historical vacuum. For example, through an anthropological filter, we find that many of the great mathematicians studied in today's classrooms are European, from a particular social class, with particular liberties and licenses, from particular seats of regional or world power. What might emerge from holistic proof thinking, which requires us to first explore the historical context of a particular mathematical contribution, is the recognition that today's educational injustices create a limited, hegemonic tale of math. Through proof thinking, we may find ourselves asking what part of math history hasn't been told – from what civilization, and from what culture of people. Through this prerequisite approach to critical proof thinking, math becomes as much a study of the humanities as it does the sciences, and so becomes more intrinsically connected to a world that is both scientific and artistic – if they are not, in fact, one and the same. Students can begin to rightfully

**EN BREF** Le rôle des preuves dans l'enseignement des mathématiques a pour fonction de développer la pensée critique, tant dans la salle de classe qu'à l'extérieur. C'est une approche analytique qui aiguise la capacité des élèves à réfléchir de façon critique et qui ne leur permet pas de formuler une réponse sans d'abord comprendre les tenants et les aboutissants de la question, même si cela est difficile. Si on lui accorde un soutien sérieux, cette approche aura des conséquences bénéfiques bien au-delà de la salle de classe. La recherche des preuves oblige les élèves et les enseignants à déconstruire et à reconstruire la manière dont se fait l'acquisition du savoir.

associate different world civilizations with a new sense of admiration, or disapproval, for they now can see one more dimension of their existence – their contribution, for better or worse, to math.

For all these reasons, students should become more passionately and rightfully involved in a subject that many fear is becoming far too technical and rigidly formulaic. Math can be about people and human connections. Perhaps it can, through proof thinking, arouse that ever-increasing, creative and epistemological curiosity that the late Paulo Freire speaks of, leaving our students and ourselves with a clear responsibility to construct and reconstruct how we come to know. |

FAYAZ NOORMOHAMED RECENTLY OBTAINED HIS B.ED. IN MATH AND PHYSICS FROM THE ONTARIO INSTITUTE FOR STUDIES IN EDUCATION OF THE UNIVERSITY OF TORONTO (OISE/UT). HE IS PRESENTLY WITH AGA KHAN EDUCATION SERVICE (AKES) IN KHOROG, TAJIKISTAN, WHERE HE IS WORKING WITH MATH AND PHYSICS TEACHERS TOWARDS TEACHER EDUCATION AND PROFESSIONAL DEVELOPMENT ACTIVITIES. FNOORMOH@ALUMNI.UWATERLOO.CA.










**Cultivate Your Commitment to Canada**  
**Cultiver son attachement au Canada**

*Spread peace   Semer le respect   Nurture belonging   Être citoyen du monde   Propager la paix   Plant respect   Nourrir l'appartenance   Be a global citizen!*

---

### New Activity Guide: Global Citizenship

**Sharing the Harvest** examines Canadian citizenship within the larger global context. It helps youth to understand the interconnectedness of people, the environment and individual action, as well as to consider the rights and responsibilities associated with being a "world citizen". The materials are designed for use in grades 4-8.



**Order your free copies online at:**  
[www.cic.gc.ca/english/citizen/belonging/onlineform.html](http://www.cic.gc.ca/english/citizen/belonging/onlineform.html)  
or call **Classroom Connections** at 1-888-882-8865.

The activity guide and the accompanying posters are products of the Promotion Division of the Integration Branch, Citizenship and Immigration Canada (CIC), and were developed in conjunction with Classroom Connections.



Citizenship and Immigration Canada



Citoyenneté et Immigration Canada



CLASSROOM CONNECTIONS

### Nouveau guide d'activités : citoyenneté globale

**La moisson** examine le concept de la citoyenneté canadienne dans son contexte le plus global. Cela aide les jeunes à comprendre les liens d'interdépendance des gens, de l'environnement et des actions individuelles, tout en abordant le sujet des droits et responsabilités associés avec le fait d'être un « citoyen de la Terre ». La ressource est conçue pour être utilisée dans la 4<sup>e</sup> à la 8<sup>e</sup> année.

**Comment puis-je obtenir des copies?**  
Rendez-vous sur le site [www.cic.gc.ca/francais/citoyen/appartenance/form-enligne.html](http://www.cic.gc.ca/francais/citoyen/appartenance/form-enligne.html) pour commander des copies ou appelez le Chaînon scolaire au 1-888-882-8865.

Ce guide d'activités et les affiches qui les accompagnent sont des produits de la division de la promotion de la Direction générale de l'intégration de Citoyenneté et Immigration Canada, et ont été développés en collaboration avec le Chaînon scolaire.



Citoyenneté et Immigration Canada



Citizenship and Immigration Canada



LE CHÂNON SCOLAIRE