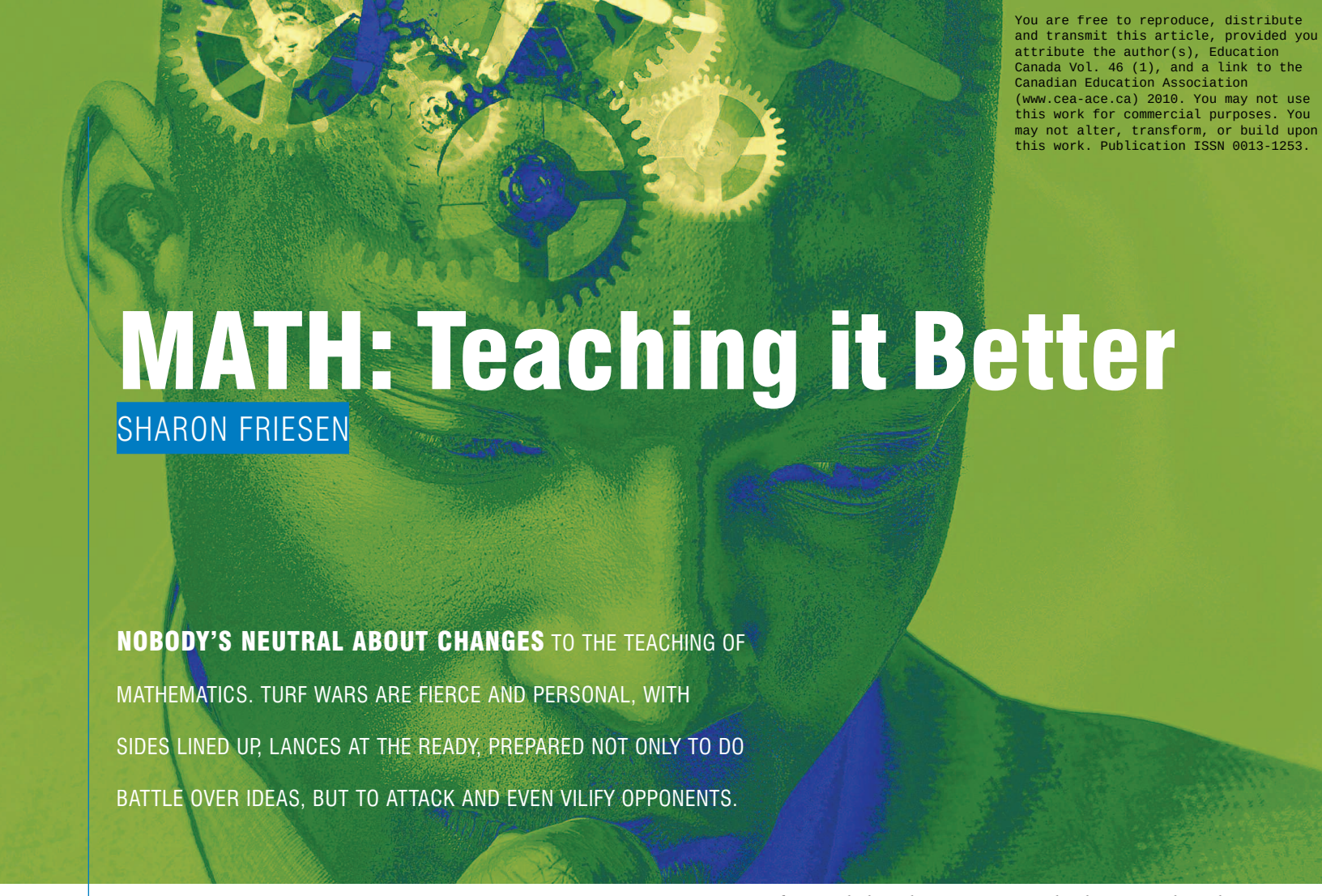


MATH: Teaching it Better

SHARON FRIESEN

NOBODY'S NEUTRAL ABOUT CHANGES TO THE TEACHING OF MATHEMATICS. TURF WARS ARE FIERCE AND PERSONAL, WITH SIDES LINED UP, LANCES AT THE READY, PREPARED NOT ONLY TO DO BATTLE OVER IDEAS, BUT TO ATTACK AND EVEN VILIFY OPPONENTS.

MANY YEARS AGO A TRUSTEE FOR A LARGE CANADIAN school district took the Board statistician to task: why were math results on provincial examinations lower than those of a neighboring jurisdiction? Frustrated by what he perceived to be a long-winded analysis, he cut the statistician off. "Look," he said, "what I want is simple. I want all the students in this district to score above average on all these exams – and if they don't, I want to know the reason why."

That's an odd little story, good for a chuckle at the trustee's mathematical expense. But it is also telling. People worry about math, even when they don't fully understand what all the fuss is about. Politicians, policy makers, teachers and parents pore over local, provincial, national and international test results in the ways old soothsayers scrutinized the entrails of chickens to assess the alignment of critical forces for future success and disaster. They want to know who's making the grade, who's not – and what can be done to make sure *their* students are the ones on top.

People believe mathematics is important. It is taken for granted that numeracy is one of the basics of a sound education. Poor math skills are a worry all over the world, and there is growing consensus around a few key beliefs:

- math "skills" involve more than memorization of number facts and formulae
- teaching for understanding is important
- being good at math means being able to solve problems

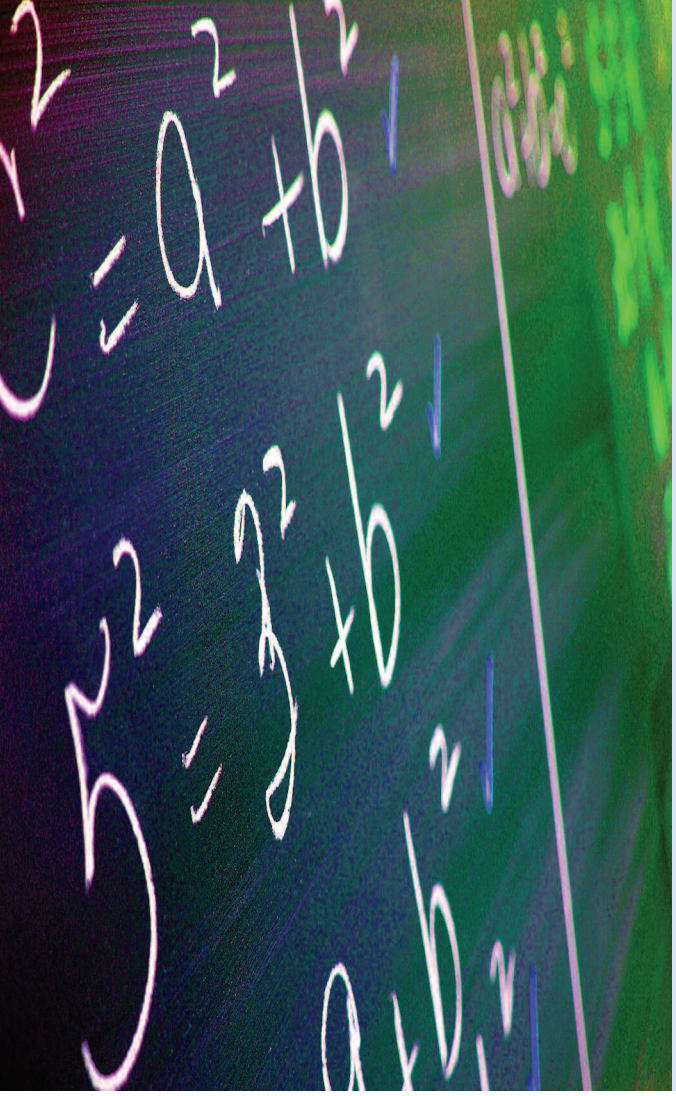
The difficulty is that there is little agreement about what all these fine statements actually mean. *Why*, precisely, is mathematics important? What should a fully numerate person in today's society be able to do? What mathematics should be taught in school, and how should it be done? If competence involves more than quick recall of number facts and algorithms on command, what precisely is that *more* that we should be teaching? What does understanding in mathematics actually look like? And does the ubiquitous dilemma of one train leaving from Chicago and another leaving from Toronto really count as a mathematically worthwhile problem anyhow?

MORE OF THE SAME

These are pressing questions in the current climate of mathematics reform. A number of large international research studies (TIMSS 1995,1999, 2003 and PISA 2003) have helped the mathematics education community address the task of improving mathematics learning and achievement for all students through teaching better mathematics and teaching mathematics better.

But the contemporary urgency of this task should not blind us to how old the fundamental questions of mathematics reform actually are. Setting these new initiatives in their historical context may help us understand the complexity of mathematics reform – and this "long view" may also help us appreciate why easy answers are usually wrong.

The first significant reform in Western mathematics teaching and learning was led by Gerbert d'Aurillac more than a thousand years ago. He believed that students needed more than explanations and lectures in order to understand abstract mathematical concepts, and so his teaching relied heavily on the use of teaching aids and manipulatives he designed and constructed to meet needs that he perceived. It is said that he "broke with all tradition by devising charts, models and instruments for demonstration to his students and for handling by them..."[1] The use of such devices had been around since the time of Quintilian, a



EN BREF **Plusieurs grandes études internationales ont aidé le milieu de l'enseignement à s'atteler à la tâche pour améliorer les programmes et les méthodes d'enseignement en mathématiques. Or, l'urgence actuelle de cette tâche ne peut nous empêcher de constater que les efforts de réforme des programmes de mathématiques ne datent pas d'hier. Déjà, il y a plus de mille ans, les tenants de diverses doctrines pédagogiques menaient des combats intellectuels acharnés au cours desquels ils n'hésitaient pas à attaquer et à vilipender leurs adversaires. La réforme des « nouvelles mathématiques » des années 1960 et le mouvement du « Retour aux sources » ne sont que les exemples les plus récents de ces guerres d'idées. Les réformes actuelles qui s'appuient sur les normes proposées par le US National Council of Teachers of Mathematics s'avèrent prometteuses puisqu'elles reconnaissent le fait que tout changement substantiel doit prendre sa source dans la salle de classe et mettre l'accent sur le perfectionnement professionnel.**

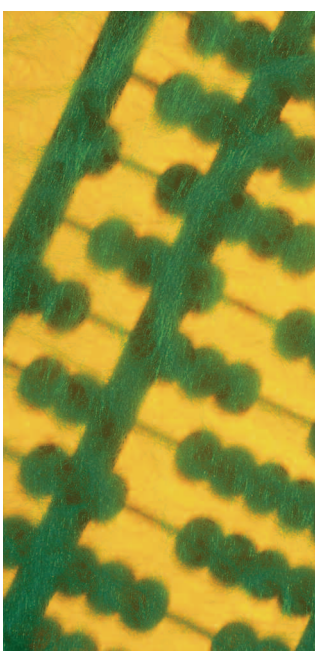
Roman educator living in the first century, but "Gerbert's innovations extended far beyond the utilization of teaching aids".[2] He used models to assist learners to grapple with abstract and difficult ideas, particularly difficult and messy mathematical ideas. And he was an early supporter of what others have come to call "learning by discovery".

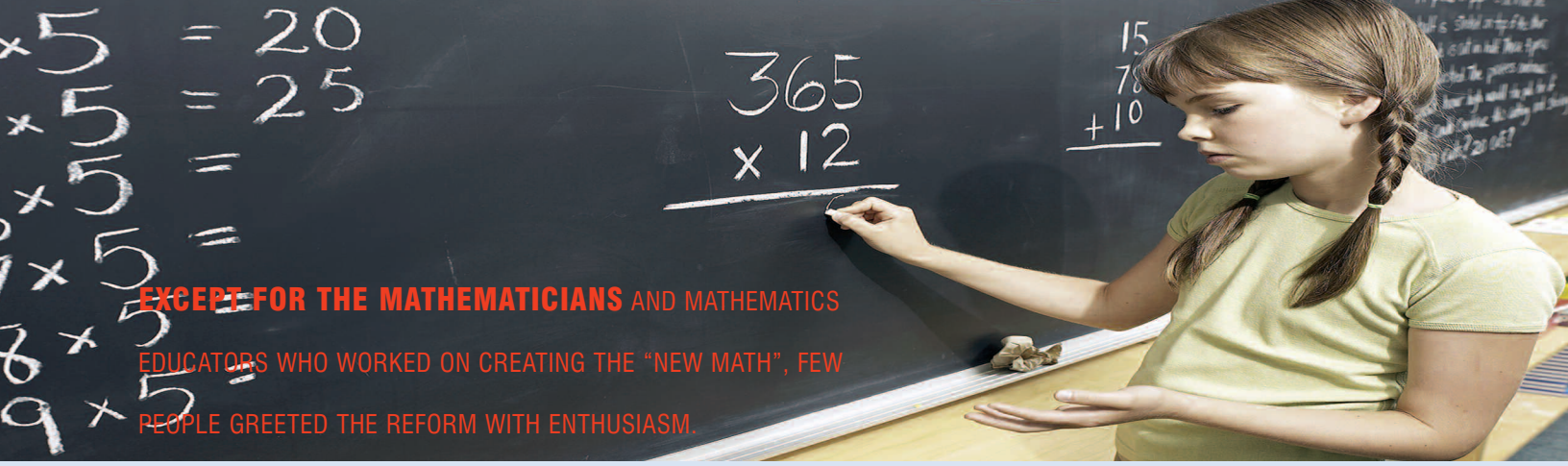
Gerbert was dedicated to improving his students' understanding; he was an exceptional pedagogue who was fully attentive to his students' increasing ability to reason and explore in mathematically sound and creative ways. Because of his success, many of his contemporaries attempted to copy his methods and devices. However, unwilling or unable to learn the pedagogy required to foster student learning and understanding with these technologies, they failed to produce the learning and deep understanding that Gerbert's own students experienced. The result was "poor instruction at best, or dissemination of incorrect information at worst".[3] What started as creative and effective new ways of teaching mathematics ended in a failed bandwagon, undermined by "ignorance and suspicion [that] helped ensure that Gerbert's innovations were not used or adopted.[4]

In 1100, William of Malmesbury declared that Gerbert's devices, early examples of the use of new technologies in the classroom, were "the work of the devil" – somewhat bewildering since in 999 AD Gerbert d"Aurillac had become Pope Sylvester II.

Three lessons emerge from Gerbert's unfortunate experience:

1. Among all subjects taught in our schools, mathematics seems to present a unique problem. There is no other discipline in which the gap between the curriculum presented to students and the body of knowledge that comprises the discipline is so great. Gerbert was a solitary and puzzling voice in calling for students to understand mathematics, and not just perform routine computational tasks presented to them. His insistence in immersing students in complex, messy, difficult mathematical problems was strikingly different from the pedagogy of colleagues – and, indeed, of much contemporary teaching in Canadian schools. Ask anyone on the street what they think mathematics is. You won't wait long until someone says it is a logical discipline where you have to learn basic things first in order to prepare for more difficult math later on. Adding and subtracting are easier than multiplication, and algebra is way too hard for little kids. Being good at math means getting right answers, fast. And you have to memorize a lot. It is unlikely that many would use words like "messy", "creative" or "imaginative" in respect to mathematical thinking – even though wonderful math reformers have been trying to introduce these ideas for a millennium.
2. The secret of teaching mathematics well does not lie in borrowing teaching strategies holus bolus. Even those of Gerbert's colleagues who admired his results and tried to emulate his success in their own classroom had terrible difficulties. There is a simplistic and all-too-common view of curriculum as something to be "delivered". Teaching, it is felt, means just adopting better delivery methods. But Gerbert's peers found it almost impossible to do that – and teachers today have the same problem with methods imposed from away. Even when they are anxious to improve their own teaching, it is not immediately clear what kind of support they need in order to keep good ideas from degenerating into failed experiments.
3. Nobody's neutral about changes to the teaching of mathematics. Turf wars are fierce and personal, with sides lined up, lances at the ready, prepared not only to do battle over ideas, but to attack and even vilify opponents. "All's fair in love and war" should also include the climate into which mathematical reformers must move. While few might be as colorful as William of Malmesbury in dismissing serious efforts at reform as diabolical, the history of attempts to change the mathematics taught in school, and how that mathematics is taught, exacts a considerable toll on those willing to wade into the fray. Nowhere, it seems, is the status quo defended with such fervor.





EXCEPT FOR THE MATHEMATICIANS AND MATHEMATICS EDUCATORS WHO WORKED ON CREATING THE "NEW MATH", FEW PEOPLE GREETED THE REFORM WITH ENTHUSIASM.

FAST FORWARD

Most of us have heard about "the new math"—a short-lived effort to reform mathematics in the wake of the Soviet launch of Sputnik on October 4, 1957. The United States, humiliated by the Soviet victory in space, launched a major math reform. "Over the ensuing months, as policy makers stopped blaming other policy makers, they began to argue that our apparent lag in science and mathematics was, in part, the product of an outdated school curriculum in those subjects. Greatly increased amounts of government money quickly became available to reform school mathematics and science, and a new era began."[5] The new math reforms were intended to bring mathematics education in K-12 more in step with the mathematics that was taught in the universities. "Because the university mathematicians who dominated the modern mathematics movement tended to be specialists in pure rather than applied mathematics, they saw pure mathematics...not only as the content that was missing from the school curriculum but also as providing the framework around which to reorganize that curriculum."[6]

So what was new about the "new math"? Well, one thing was a switch from teachers' telling to students' learning through investigation, discovery, and hands-on learning. Sound familiar?

Math educators once again tackled the problem of teaching abstract mathematical ideas in ways that would develop sound understanding.

> One focus of the new math was set theory, where students were encouraged to think of numbers in a new, hopefully more concrete way. Students would take a set of four items, and add it to another set of five. Yes, the result was still nine, but the emphasis was on the concept of addition, rather than the answer per se. Using this technique, students were hoped to discover that the sets would yield the same number regardless of their order (the commutative property), and that taking one original set from the combined set would yield the other original set, thereby discovering subtraction, the inverse of addition. Other aspects of the new math including using number bases other than base-10 and introducing more abstract number theory concepts such as prime numbers earlier in the students' careers.[7]

Before the ink was dry on the new math reforms, mathematicians started to line up on either side of a fierce debate. Morris Kline, a preeminent mathematician and an ardent critic of the math reforms, led one charge. He believed that the efforts of the reform were "'wholly misguided,' 'sheer nonsense,' attempts to replace the 'fruitful and rich essence of mathematics' with sterile, peripheral, pedantic details".[8]

Except for the mathematicians and mathematics educators who worked on creating the "new math", few people greeted the reform with enthusiasm. Teachers ran into problems trying to teach the new curriculum. Parents, too, had difficulty with it, frustrated that they could no longer help even their second graders with their homework. Hands-on mathematics just didn't look like real math to teachers and parents who had learned so much of their own mathematics by rote. By 1965 the new math reform had already disappeared from most Canadian classrooms, and in 1970 the U.S. National Science Foundation withdrew funding from the reform initiative, thereby ending it officially.

During the late 1960's, 1970's and 1980's, "back to the basics" quickly replaced the failed 'new math' curriculum, returning mathematics education to familiar approaches that everyone recognized: rote recitation of facts, memorization of algorithms, and solving of routinized word problems, As familiar as these conventional approaches to the teaching of mathematics were to teachers and parents, they left essentially untouched the actual problem of teaching better mathematics, better. A 1983 report, *A Nation At Risk*, signaled the lack of satisfactory outcomes and student achievement from this conventional approach, and mathematics reformers were back at the drawing board again. This time it was the mathematics educators who stepped forward, and what began as a grass roots movement turned into a mathematics reform based on standards articulated by the US National Council of Teachers of Mathematics (NCTM).[9]

CURRENT REFORM INITIATIVES

The principles and standards articulated by NCTM provided the basis for the development of many mathematics curricula worldwide, including Canada's.

> *Principles and Standards for School Mathematics* describes a future in which all students have access to rigorous, high-quality mathematics instruction, including four years of high school mathematics. Knowledgeable teachers have adequate support and ongoing access to professional development. The curriculum is mathematically rich, providing students with opportunities to learn important mathematical concepts and procedures with understanding. Students have access to technologies that broaden and deepen their understanding of mathematics. More students pursue educational paths that prepare them for lifelong work as mathematicians, statisticians, engineers, and scientists.
>
> This vision of mathematics teaching and learning is not the reality in the majority of classrooms, schools, and districts. Today, many students are not learning the mathematics they need. In some instances, students do not have the opportunity to learn significant mathematics. In others, students lack commitment or are not engaged by existing curricula. — *National Council Of Teachers of Mathematics*

While NCTM Standards and Principles have had both detractors and supporters, they appear to have developed a staying power that earlier reform efforts lacked, and that have provided a base on which findings from international testing and research are now building.

LEARNING TO IMPROVE

Lessons learned from these latest reform efforts are only now emerging as tentative findings rather than as prescriptive recipes for change. Among the most promising are these:

1. Not all teaching methods are equal. While every country has its own way of teaching mathematics; there are features that exemplify methods of teaching that help students achieve well.[10]
2. When relationships between facts, procedure, concepts and problem solving are attended to and made explicit in the mathematics classroom, student achievement increases.[11]
3. When students grapple with and solve challenging problems, their mathematics achievement increases.[12]
4. Reform recommendations disseminated through curriculum documents and delivered to teachers are insufficient to improve the teaching and learning of mathematics. So is simply issuing edicts that teachers will be more accountable. However necessary solid standards, excellent curricula, innovative materials and accountability, they are not sufficient to ensure that teachers increase students' mathematical understanding and achievement. Nor do they ensure quality math teaching for all students.
5. Mechanisms must be built into the teaching profession to permit teachers "to improve gradually over time."[13]

IMPROVING OVER TIME

One of the most promising differences between earlier reform efforts and those being developed today is the emphasis on professional development. Meaningful mathematical reform must start with the classroom. If change is to take hold, teachers and students alike have to accept, understand and enact them within the context of what they already know about mathematics. Thus, new ways of working cannot be so alien as to be unrecognizable to teachers. Rather, they should connect in meaningful ways with what teachers already know, and then work to extend that knowledge in significant ways.

WE HAVE COME TO UNDERSTAND THAT REASONING AND PROBLEM SOLVING HAVE TO BE THE FOCUS OF MATHEMATICS INSTRUCTION IN THE CLASSROOM...REASONING AND PROBLEM SOLVING REQUIRE STUDENTS TO LEARN HOW TO GENERATE, EVALUATE, JUSTIFY AND REVISE MATHEMATICAL MODELS WHILE SOLVING PROBLEMS.

What teachers know and understand about mathematics makes a difference to the quality of their teaching. This means more professional development dedicated to

- improving teachers' knowledge of mathematics,
- improving the ways in which they know and understand content so they can teach it better, and
- improving their knowledge of how students learn particular concepts and topics.

And finally, we have come to understand that reasoning and problem solving have to be the focus of mathematics instruction in the classroom. This last point is important and tied strongly to all the others. Reasoning and problem solving require students to learn how to generate, evaluate, justify and revise mathematical models while solving problems. They also require teachers to involve students in activities that require generalizing and justifying various solutions to problems. The teaching practices that embody modeling, generalization and justification in order to sponsor mathematical reasoning and problem solving represent a fundamental shift in the ways that many teachers now teach mathematics – although Gerbert would probably have found such practices exciting.

CONCLUSION

History has shown us that mathematics reforms of the past have never quite lived up to what their creators and supporters wanted, nor to what their detractors and opponents feared. While the task of creating classrooms in which students understand abstract and difficult mathematical ideas, see relevance in the mathematics they are learning, and achieve mathematical competence seems daunting, as a mathematics community we are further down the road in knowing what to do to achieve these goals. We have made demonstrable progress by working together – mathematicians, mathematics educators, and

teachers who understand that mathematics reform is a complex matter. There are no easy answers.

As we move our efforts into the place where people learn, and teach, and do mathematics – the classroom – we also need to be mindful and open to the our next question: Are we teaching the right mathematics content to achieve goals of numeracy or qualitative literacy required for meaningful participation in democratic societies?

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Notes

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