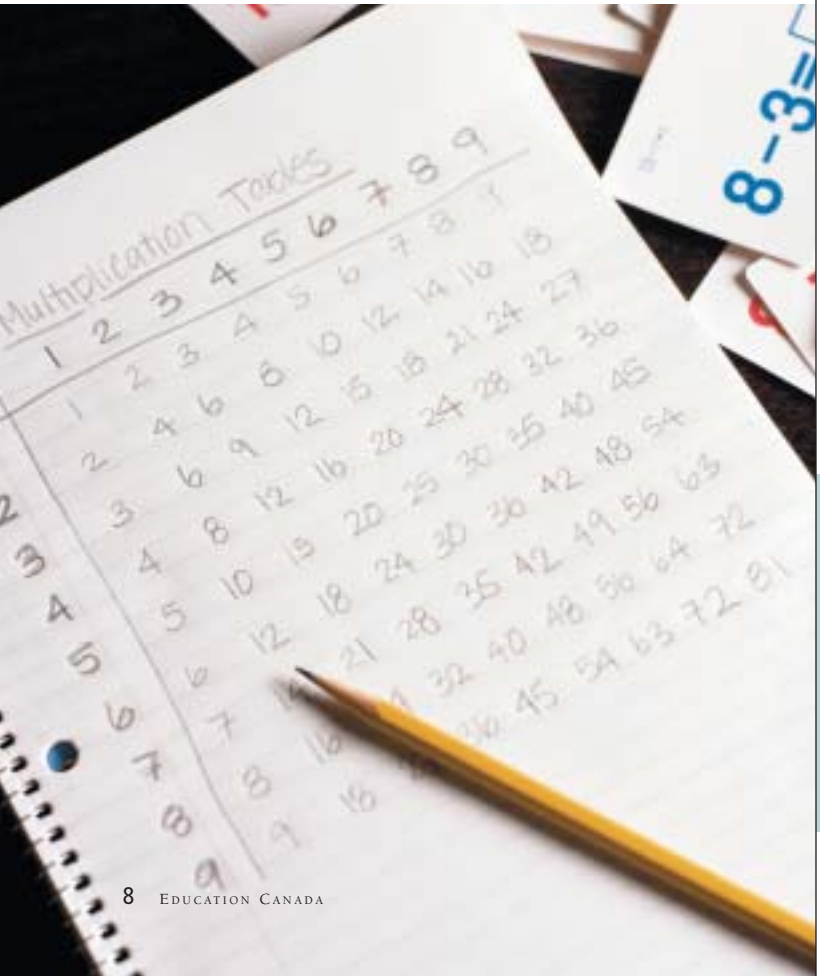
Teaching and Understanding Mathematics

Leo Jonker

In the last quarter century, we have spent considerable energy attempting to improve student mathematics outcomes, including improved curricula at all levels, much of it inspired by the 1989 publication of the Curriculum and Evaluation Standards for School Mathematics by the National Council of Teachers of Mathematics (NCTM). And yet, the results do not seem commensurate with the efforts. One reason – almost certainly – is the extent to which teachers themselves do not understand or like mathematics. Students who choose to go into elementary education tend to specialise in the humanities or the social sciences. Many gave up on mathematics some time ago, and even fear the subject.



Too often, the courses offered at University do not help. They assume that students have taken high school mathematics and have a good understanding of the subject at that level. Most prospective elementary teachers avoid these courses and, when mathematics is required for their programmes, carefully select the ones that look easiest.

As a result, they begin teaching with very little sense of mathematics – even at the elementary level – as a structured subject connected by explanations and powerful concepts. Especially now that the NCTM-inspired curriculum places tremendous stress on understanding, teachers need to understand the connections between the parts and have a sense of the over-all structure of mathematics at the level they teach. Without it, they cannot judge the accounts given by bright and lively pupils, nor provide closure to a discussion about a mathematical procedure. Without it they are tempted to fall back on textbook learning and transmission models of teaching.

The prospective teachers I deal with are painfully aware of this weakness and are keen to find a better way. One of them wrote, "I really dislike the fact that I feel as though I have just squeaked through math all my life rather than really understanding it. Although I have always gotten good marks in the math courses I have taken, I do not feel comfortable with the subject in any way."

Not every academic mathematician has the time or the inclination to become involved in mathematics education at the elementary school level. My interest began when my own children were elementary school pupils. Resources were scarce, especially for enrichment opportunities, so I began to offer an enrichment class for pupils in grades 6-8. I enjoyed it so much that I have been doing it (with a few interruptions) ever since, gradually developing a two-year cycle for students in grades 7 and 8, focusing on on number patterns, algebra, and probability in year one and geometry in year two. The programme has

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La plupart des nouveaux enseignants commencent à enseigner les mathématiques sans vraiment comprendre qu'il s'agit d'une matière structurée qui s'appuie sur un ensemble de notions dynamiques. Les programmes universitaires de mathématiques présupposent — souvent à tort — que les nouveaux étudiants et étudiantes ont au départ une bonne compréhension de la matière. M. Jonkers décrit un nouveau cours de mathématiques, offert à l'Université Queens, qui utilise les modules d'enseignement usuels de l'école et la salle de classe universitaire pour aider les enseignants et enseignantes de niveau primaire à voir les mathématiques comme un tout organique.

now been compiled as a two-volume manual written for others who might like to teach a similar programme.

Given the widespread concern about elementary school mathematics education, I decided to apply my elementary school enrichment experience to the post-secondary level by offering a new mathematics course for prospective elementary school teachers at Queen's University.

After discussing some of the material in the enrichment manuals – number patterns, infinite sets, conjectures, proofs, growth of sequences, prime factors, fractions and repeated decimals, irrational numbers, and divisibility tricks – the students are sent out in pairs to local elementary schools to offer a ten week long, once-a-week enrichment class for pupils in grades 7 and 8. The students submit written reports on these weekly visits in which they reflect on their teaching experiences and on the way the lessons fit into the provincial mathematics curriculum. We also use class time and assignments to discuss a variety of other, more basic, issues around elementary mathematics: Why does long division work the way it does? Why do we invert and multiply when we divide by a fraction? How would you make the commutative law of multiplication clear to elementary pupils, or the distributive law? In what way is the pedagogical value of one method greater than another?

I think the course works particularly well for prospective teachers for a number of reasons:

- The enrichment material is sufficiently challenging to engage all the students in the course. It does not try to advance elementary pupils beyond their grade levels, but rather looks more deeply and reflectively into the standard curriculum. This means that students taking the university course are not tempted to mechanistically apply what they learned in high school to the questions raised.

- Students work very hard to learn the class material when they know they are required to teach it to young people later. Much of the time spent preparing their lesson plans is time spent learning new and deep mathematics. One student writes "I have never felt like I have been benefiting as much from a class as I am in this one. I love it! It is perfect for concurrent education students and anyone who plans on teaching."

- Students realize that they will never teach good mathematics lessons unless they themselves feel comfortable with the subject. One expresses it this way: "This course is fantastic. It is giving me more confidence to teach math – which is so important since I will have to be doing that soon!"

- Students need a chance to work with mathematics in a way that stresses concepts over rote learning and algorithms. In the words of one of the students "The course material is challenging in the sense that it forces me to approach math in a way that I'm not used to doing."

Although I have not yet completed a survey of the principals of the schools involved, the weekly reports from the schools also indicate a high degree of satisfaction. One student, reflecting on her visits, writes "In general, I think we are instilling a more questioning nature in students, as they are starting to ask why we do things the way we do in math rather than just accepting everything that is put before them."

This year the course is very much an experiment. However, already interest runs high. Local schools receive a ten-week focused enrichment programme. University students get teaching experience with a small group of interested and generally well-behaved pupils. And those for whom mathematics has been a disconnected jumble of formulas and problem types have a chance to see the connections, to enjoy some of the beauty, and to develop their sense of mathematics as an organic whole.

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